Course 11

Wednesday, 16 October 2024 09:57

EE557Cours 11 -...

EPFL

Génie Electrique et Electronique
Master Program
Prof. Elison Matioli

EE-557
Semiconductor devices I

Continuity and Shockley equations

Outline of the lecture

- Continuity equations
- Shockley equations
- Examples and exercises

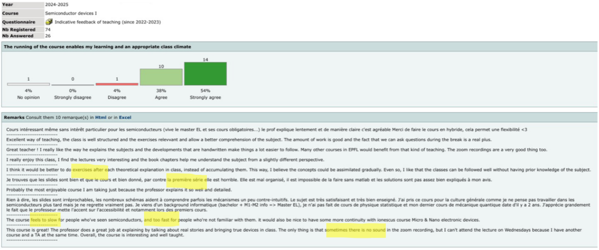
Read Chapter 5 (from Shockley equations) of the reference book (on moodle)

References:

- J. A. del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/)

Course feedback

EPFL



Year	2024-2025
Course	Semiconductor devices I
Questionnaire	Indicative feedback of teaching (since 2022-2023)
Nb Registered	74
Nb Answered	26

The running of the course enables my learning and an appropriate class climate

Remarks Consult them 10 remarque(s) in Html or in Excel

Cours intéressant même sans intérêt particulier pour les semiconducteurs (vive le master EL et ses cours obligatoires...) le prof explique lentement et de manière claire c'est agréable Merci de faire le cours en hybride, cela permet une flexibilité <3

Excellent way of teaching, the class is well structured and the exercises relevvant and allow a better comprehension of the subject. The amount of work is good and the fact that we can ask questions during the break is a real plus.

Great teacher ! I really like the way he explains the subjects and the developments that are handwritten make things a lot easier to follow. Many other courses in EPFL would benefit from that kind of teaching. The zoom recordings are a very good thing too.

I really enjoy this class, I find the lectures very interesting and the book chapters help me understand the subject from a slightly different perspective.

I think it would be better to do exercises after each theoretical explanation in class, instead of accumulating them. This way, I believe the concepts could be assimilated gradually. Even so, I like that the classes can be followed well without having prior knowledge of the subject.

Je trouves que les slides sont bien et que le cours et bien donné, par contre la première série elle est horrible. Elle est mal organisé, il est impossible de la faire sans matlab et les solutions sont pas assez bien expliqués à mon avis.

Probably the most enjoyable course I am taking just because the professor explains it so well and detailed.

Rien à dire, les slides sont irréprochables, les nombreux schémas aident à comprendre parfois les mécanismes un peu contre-intuitifs. Le sujet est très satisfaisant et très bien enseigné. J'ai pris ce cours pour la culture générale comme je ne pense pas travailler dans les semiconducteurs plus tard mais je ne regrette vraiment pas. Je viens d'un background informatique (bachelor + M1-M2 info => Master EL), je n'ai pas fait de cours de physique statistique et mon dernier cours de mécanique quantique date d'il y a 2 ans. J'apprécie grandement le fait que le professeur mette l'accent sur l'accessibilité et notamment lors des premiers cours.

The course feels to slow for people who've seen semiconductors, and too fast for people who're not familiar with them. it would also be nice to have some more continuity with ionescus course Micro & Nano electronic devices.

This course is great! The professor does a great job at explaining by talking about real stories and bringing true devices in class. The only thing is that sometimes there is no sound in the zoom recording, but I can't attend the lecture on Wednesdays because I have another course and a TA at the same time. Overall, the course is interesting and well taught.

Key questions

So far we have learned:

- Recombination
- Generation
- Drift
- Diffusion

How to describe carrier flow in semiconductors taking into account all these mechanisms?

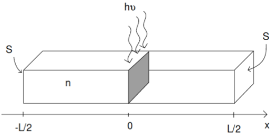
Carrier flow

Carrier dynamics:

$$\frac{dn}{dt} = \frac{dp}{dt} = G - R$$

Applies only to uniform problems in space!

This doesn't work in problems like this: Generation of carriers in a given position of space



Equation system does not capture:

- **Impact of carrier movement** on carrier concentration
  (i.e. when carriers move away from a point, their concentration drops!)
- **Boundary conditions** (surfaces are not infinitely far away – covered in Del Alamo-Chapter 5)

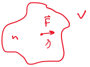
We need to include the flow of carriers

Carrier flow

Continuity equations:

For electrons:
$$\underbrace{\frac{\partial n}{\partial t}} = \underbrace{G - R} + \underbrace{\frac{1}{q}\vec{\nabla}.\vec{J_e}}$$

For holes:
$$\frac{\partial p}{\partial t} = G - R - \frac{1}{q}\vec{\nabla}.\vec{J_h}$$



$$\iint \vec{F}.d\vec{S}$$

$$\vec{\nabla}.\vec{F} = \left(\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}\right)$$

Carrier flow

Semiconductor physics so far: Shockley's equations

Gauss' law: $\underline{\vec{\nabla}\cdot\vec{\mathcal{E}}} = \frac{q}{\epsilon}\underline{(p - n + N_D^+ - N_A^-)}$

Electron current equation: $\underline{\vec{J_e}} = -qn\underline{\vec{v}_e^{drift}} + qD_e\vec{\nabla}n$

Hole current equation: $\underline{\vec{J_h}} = \underbrace{qp\vec{v}_h^{drift}}_{\text{drift}} - \underbrace{qD_h\vec{\nabla}p}_{\text{diffusion}}$

Electron continuity equation: $\underline{\frac{\partial n}{\partial t}} = G_{ext} - U(n,p) + \frac{1}{q}\vec{\nabla}\cdot\vec{J_e}$

Hole continuity equation: $\underline{\frac{\partial p}{\partial t}} = G_{ext} - U(n,p) - \frac{1}{q}\vec{\nabla}\cdot\vec{J_h}$

Total current equation: $\boxed{\vec{J_t} = \vec{J_e} + \vec{J_h}}$

System of non-linear, coupled partial differential equations: generally not solvable in a closed form.

*** Attention: Here the net recombination rate *U(n,p)* includes all the absolute net recombination rates, including surface recombination, which were not discussed in this course.

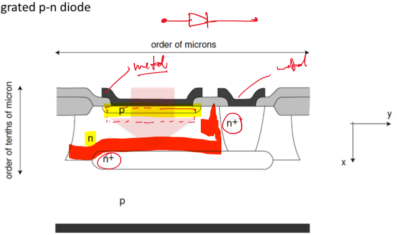
Simplifications of Shockley equations to 1D quasi-neutral situations

One-dimensional approximation

1D approximation: $\vec{\nabla} \Rightarrow \frac{\partial}{\partial x}$

In many cases, complex problems can be broken into several 1D sub-problems.

Example: integrated p-n diode



Shockley equations in 1D quasi-neutral situations

in 1D:

Gauss' law: $\frac{\partial \mathcal{E}}{\partial x} = \frac{q}{\epsilon}(p - n + N_D - N_A)$

Electron current equation: $J_e = -qnv_e^{drift}(\mathcal{E}) + qD_e\frac{\partial n}{\partial x}$

Hole current equation: $J_h = qpv_h^{drift}(\mathcal{E}) - qD_h\frac{\partial p}{\partial x}$

Electron continuity equation: $\frac{\partial n}{\partial t} = G_{ext} - U(n,p) + \frac{1}{q}\frac{\partial J_e}{\partial x}$

Hole continuity equation: $\frac{\partial p}{\partial t} = G_{ext} - U(n,p) - \frac{1}{q}\frac{\partial J_h}{\partial x}$

Total current equation: $J_t = J_e + J_h$

Equation set difficult because of coupling through Gauss' law

Two broad classes of important situations to break Gauss' law coupling:

1. Carrier concentrations are high: **quasi-neutral situation**
   - majority carrier concentration nearly tracks the doping concentration
   - Any introduction of extra carriers in a volume is negligible compared to the charge density already present

$$\rho \simeq 0 \Rightarrow \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

2. Carrier concentrations are very low: **space-charge** and **high-resistivity situations**: **E independent of n, p**

Simplifications of Shockley equations to 1D quasi-neutral situations

Quasi-neutral (QN) approximation

At every location, the net volume charge that arises from a discrepancy of the concentration of positive and negative species is **negligible** in the scale of the charge density that is present.

$$\rho \simeq 0 \;\Rightarrow\; \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

QN approximation eliminates Gauss' law from the set:

$$\rho = q\underbrace{(p - n + N_D^+ - N_A^-)} = q\underbrace{(p_o - n_o + N_D^+ - N_A^-)}_{\text{equilibrium}} + q\underbrace{(p' - n')}_{\text{extra}}$$

Quasi-neutrality out of equilibrium:

$$|\frac{p'-n'}{n'}| \simeq |\frac{p'-n'}{p'}| \ll 1$$

$$p' \simeq n' \qquad \Rightarrow \boxed{\rho = 0}$$

QN approximation is good if **n, p high** ⇒carriers move to erase ρ.

Simplifications of Shockley equations to 1D quasi-neutral situations

Consequence of quasi-neutrality

$$\rho \simeq 0 \;\Rightarrow\; \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

$$\underline{\mathcal{E}} = \underline{\mathcal{E}_o} + \underline{\mathcal{E}'}$$

Then, in equilibrium:

$$\underline{\frac{\partial \mathcal{E}_o}{\partial x} = \frac{q}{\epsilon}(p_o - n_o + N_D^+ - N_A^-)}$$

and out of equilibrium:

$$\underline{\frac{\partial \mathcal{E}'}{\partial x} = \frac{q}{\epsilon}(p' - n')}$$

Simplifications of Shockley equations to 1D quasi-neutral situations

Consequence of quasi-neutrality

Subtract one continuity equation from the other:

(1) $$\frac{\partial n}{\partial t} = G_{ext} - U(n,p) + \frac{1}{q}\frac{\partial J_e}{\partial x}$$

(2) $$\frac{\partial p}{\partial t} = G_{ext} - U(n,p) - \frac{1}{q}\frac{\partial J_h}{\partial x}$$

(1) – (2): $$\frac{\partial J_t}{\partial x} = q\frac{\partial (n-p)}{\partial t} = -\frac{\partial \rho}{\partial t}$$

Total current must be continuous everywhere: **current continuity**

Continuity equation for net volume charge: **if J_t changes with position, ρ changes with time**.

- In *Static case*:

$$\frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\partial J_t}{\partial x} = 0,\ J_t \text{ independent of } x$$

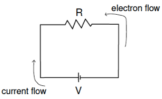
- In *Dynamic case*, we also have in most useful situations:

$$\frac{\partial \rho}{\partial t} \simeq 0 \text{ in times scale of interest}$$

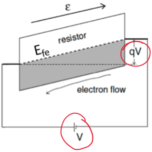
Case 1: Majority-carrier type situations

Semiconductor resistor

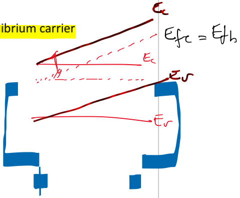
Voltage applied to extrinsic quasi-neutral semiconductor, without changing the equilibrium carrier concentrations:



circuit view



energy view



The battery picks up electrons from positive terminal, increases their potential energy and puts them at the negative terminal.

Energy given to electrons: **E = qV**

If provided with a path (resistor), electrons flow.

Case 1: Majority-carrier type situations

Semiconductor resistor

Characteristics of majority carrier-type situations:

- Small electric field imposed from outside: does not disturb the dynamic balance existing in equilibrium
- Carriers are not disturbed from equilibrium: time derivatives are zero!
- electrons and holes drift $\frac{dJ_e}{dx} \simeq 0,\ \frac{dJ_h}{dx} \simeq 0,\ \frac{dJ_t}{dx} \simeq 0$

$$\frac{\partial n}{\partial t} = 0 \Rightarrow \frac{\partial J_e}{\partial x} = 0$$

Electron and hole concentrations unperturbed from TE Simplifications:

- neglect contribution of minority carriers
- neglect time derivatives of carrier concentrations

⇒ problem becomes completely quasi-static
Equation set for 1D majority-carrier type situations:

n-type	p-type
$n \simeq n_o \simeq N_D$	$p \simeq p_o \simeq N_A$
$J_e = -qn_o[v_{de}(\mathcal{E}) - v_{de}(\mathcal{E}_o)]$	$J_h = qp_o[v_{dh}(\mathcal{E}) - v_{dh}(\mathcal{E}_o)]$
$\frac{dJ_e}{dx} \simeq 0,\ \frac{dJ_h}{dx} \simeq 0,\ \frac{dJ_t}{dx} \simeq 0$	
$J_t \simeq J_e$	$J_t \simeq J_h$

In T.E: $J_e = -q\cdot n_o\cdot v_e(\mathcal{E}_o) + q\cdot D_e\cdot\frac{dn_o}{dx} = 0$

$$\underbrace{q D_e \frac{dn_o}{dx}}_{*} = q\, n_o\, v_e(\mathcal{E}_o)$$

Out of equilibrium: $(\mathcal{E} = \mathcal{E}' + \mathcal{E}_o)$, $n \simeq n_o$, $\frac{dn}{dx} \simeq \frac{dn_o}{dx}$

$$J_e = -q\cdot n_o\cdot v_e(\mathcal{E}) + \underbrace{q\cdot D_e\cdot\frac{dn_o}{dx}}$$

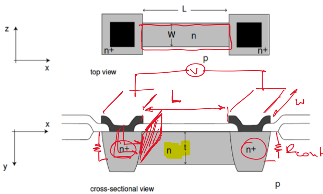
$$\boxed{J_e = -q\, n_o\cdot\left(v_e(\mathcal{E}) - \overset{*}{v_e(\mathcal{E}_o)}\right)}$$

Under low field $\vec{v}_e = -\mu_e\cdot\vec{\mathcal{E}}$

$$J_e = +q\cdot n_o\cdot\mu_e\cdot\mathcal{E}'$$

Simplifications of Shockley equations to 1D quasi-neutral situations

Integrated Resistor with uniform doping (n-type)



Under low $\mathcal{E}$

$v_e = -\mu\cdot\mathcal{E}$

$$\underset{A/cm^2}{J_e} = q\cdot n_o\cdot\mu_e\cdot\underset{V/cm}{\mathcal{E}} = V/L$$

$J_e\, x$

Equations:

$$J_t = -qN_D v_e^{drift}(\mathcal{E})$$

In general (low and high fields):

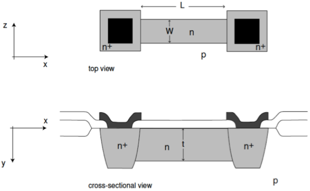
$$J_t \simeq qN_D\mu_e\mathcal{E}$$

$$I = WtqN_D\mu_e\frac{V}{L}$$

$$I = WtqN_D\underbrace{\frac{v_{sat}}{1+\frac{v_{sat}}{\mu_e}\frac{L}{V}}}$$

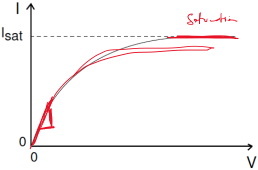
Simplifications of Shockley equations to 1D quasi-neutral situations

Integrated Resistor with uniform doping (n-type)



which for high fields saturates to:

$$I_{sat} = WtqN_Dv_{sat}$$



$$I = \underbrace{w . t}_{A} . \; q . \; N_D . \; \mu_e . \; \frac{V}{L}$$

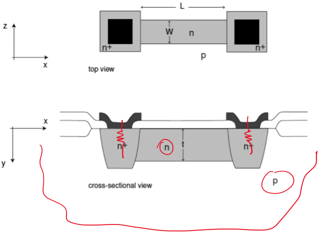
$$R = \frac{L}{q . A . \mu_e . N_D} = \rho . \frac{L}{A} \Rightarrow \rho = \frac{1}{q . N_D . \mu_e}$$

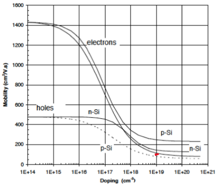
$$R = R_{sh} . \frac{L}{W} \Rightarrow R_{sh} = \frac{1}{q . \mu_e . N_D . t}$$

Exercise from list 2:

Consider an integrated resistor composed of an n-type Si layer fabricated on a p-type Si. The doping level is $N_D = 10^{19} cm^{-3}$. The dimensions of its active region are L=5um W=2um and t=0.1um. At RT, estimate:

1. Sheet resistance of the n-type semiconductor
2. Resistance of the resistor.





$$R_{sh} = \frac{1}{q \cdot \mu_e \cdot N_D \cdot t} = 570\ \Omega/\square$$

$$N_D = 10^{19}\ cm^{-3} \Rightarrow \mu_e = 110\ cm^2/Vs$$

$$R = R_{sh} \cdot \frac{L}{W} = 570 \cdot \frac{5}{2} = \underline{1.5\ k\Omega}$$

Case 2: Minority-carrier type situations

- Low level injection
- Excess carriers
- No significant electric field
- Quasi-neutrality

n-type	p-type
$p_o - n_o + N_D - N_A \simeq 0$	
$p' \simeq n'$	
$J_e = qn_o\mu_e\mathcal{E}' + qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$	$J_e = qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$
$J_h = qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$	$J_h = qp_o\mu_h\mathcal{E}' + qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$
$D_h\frac{\partial^2 p'}{\partial x^2} - \mu_h\mathcal{E}_o\frac{\partial p'}{\partial x} - \frac{p'}{\tau} + G_{ext} = \frac{\partial p'}{\partial t}$	$D_e\frac{\partial^2 n'}{\partial x^2} + \mu_e\mathcal{E}_o\frac{\partial n'}{\partial x} - \frac{n'}{\tau} + G_{ext} = \frac{\partial n'}{\partial t}$
$\frac{\partial J_t}{\partial x} \simeq 0$	
$J_t = J_e + J_h$	

n-type :

L.L.I: $n \simeq n_0$

$p \simeq p'$

n_0

p_0

$p' = n'$ very small

if n-type $n = n_0$

$p_0 = p'$

$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}'$

. recombination rates : $U = p'/\tau$

, internally generated electric field is very small

$$J_e = q.(n_0 + n').\mu_e(\mathcal{E}_0 + \mathcal{E}') + q D_e \left(\frac{\partial n_0}{\partial x} + \frac{\partial n'}{\partial x}\right)$$

in T.E : $n' \simeq 0$ $\quad \mathcal{E}' = 0$ $\quad J_e = 0$ $\Rightarrow \boxed{q.n_0.\mu_e.\mathcal{E}_0 = -q.D_e \frac{\partial n_0}{\partial x}}$ [1]

out of equilibrium:

$$J_e = q.n_0.\mu_e.\mathcal{E}' + \cancel{q.n_0.\mu_e.\mathcal{E}_0}$$

$$+ \overset{\sim 0}{\cancel{q.n'.\mu_e.\mathcal{E}'}} + q.n'.\mu_e.\mathcal{E}_0$$

$$+ \cancel{q.D_e \frac{\partial n_0}{\partial x}} \xrightarrow{[1]} = -q.n_0.\mu_e.\mathcal{E}_0$$

$$+ q.D_e \frac{\partial n'}{\partial x}$$

majority carriers:

$$J_e = \underbrace{q.n_0.\mu_e.\mathcal{E}'}_{\text{drift of } n_0 \text{ due to } \mathcal{E}'} + \underbrace{q.n'.\mu_e.\mathcal{E}_0}_{\text{drift of } n' \text{ due to } \mathcal{E}_0} + \underbrace{q.D_e.\frac{\partial n'}{\partial x}}_{\text{diffusion}}$$

minority carriers:

$$J_h = \underset{\sim 0}{\cancel{q.p_0.\mu_h.\mathcal{E}'}} + q.p'.\mu_h.\mathcal{E}_0 - q.D_h.\frac{\partial p'}{\partial x}$$

Continuity equation:

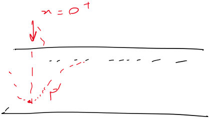
$$\frac{\partial p'}{\partial t} = \underline{G_{ext}} - \underbrace{(G_{int} - R)}_{U} - \frac{1}{\underline{q}} \frac{\partial J_h}{\partial x}$$

$$\boxed{\frac{\partial p'}{\partial t} = G_{ext} - p'/\tau - \frac{\partial p'}{\partial x} \cdot \mu_h \cdot \mathcal{E}_0 + D_h \cdot \frac{\partial^2 p'}{\partial x^2}}$$

Out of the origin:

$G_{ext} = 0$

quasi-neutral region: $\mathcal{E}_0 \simeq 0$



$x > 0$

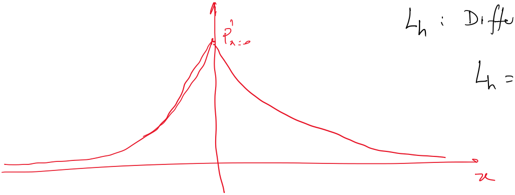
$$-\frac{p'}{\tau} + D_h \cdot \frac{\partial^2 p'}{\partial x^2} = 0$$

$$p' = \cancel{A^{=0} \exp\left(+x/L_h\right)} + B \cdot \exp\left(-\frac{x}{L_h}\right)$$

$$p' = B \cdot \exp\left(-x/\underline{L_h}\right)$$

L_h: Diffusion length

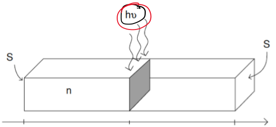
$$L_h = \sqrt{D_h \cdot \tau}$$

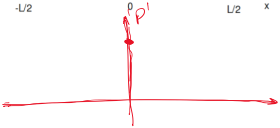


Case 2: Minority-carrier type situations

- Let's look at a n-type bar

G_{ext} : g_e : generation of minority carriers





Key conclusions

Shockley equations:
system of equations that describes carrier phenomena in semiconductors in the drift-diffusion regime.

Quasi-neutral approximation:
appropriate if semiconductor is sufficiently extrinsic: $\rho \sim 0 \Rightarrow$

$$n_o - p_o \simeq N_D - N_A \qquad n' \simeq p'$$

Majority carrier-type situations characterized by application of **external voltage without perturbing carrier concentrations**.

Majority-carrier type situations dominated by drift of majority carriers.
Integrated resistor:
- for low voltages, current proportional to voltage across
- for high voltages, current saturates due to v^{sat}